

Considerations on Neo-Fregean Ontology

Nikolaj Jang Pedersen

1. Introduction

Consider the following abstraction principle – often referred to as *Hume's Principle* in the literature –

$$(HP) \quad (\forall X)(\forall Y)(\text{Num}(X) = \text{Num}(Y) \leftrightarrow X \approx_{1-1} Y)$$

i.e. for any concepts X and Y , the number of X 's and the number of Y 's are identical if and only if there is a 1-1 correspondence between X and Y .¹ The central claim of neo-Fregeanism with respect to arithmetic is that arithmetical knowledge can be obtained a priori through Frege's Theorem, the result that the axioms of arithmetic are derivable in the system obtained by adding Hume's Principle to second-order logic.²

According to the neo-Fregean, the concept introduced by means of an abstraction principle – the concept of Number in the case of Hume's Principle – should be genuinely sortal. That is, the principle should supply a criterion of application and a criterion of identity for the relevant concept. The criterion of application tells us to what objects the concept applies, while, among these objects, the criterion of identity distinguishes one from another.

In the context of the neo-Fregean programme, the Caesar Problem arises because abstraction principles, though providing a criterion of identity for the concepts they introduce, fail to supply a criterion of application for these concepts. Hume's Principle, e.g., will tell us, for any two numbers, whether they are identical or not, but leaves us incapable of conferring a determinate truth-value upon mixed identity statements such as "Caesar is the number 0."

In recent work on the Caesar Problem,³ Bob Hale and Crispin Wright concede that abstraction principles do not succeed in supplying a criterion of application for the concepts they serve to introduce. However, they maintain that, when cashed out properly, the framework of sortal concepts and criteria of identity provides the resources needed to tell a story that will make it clear that Caesar is not – and indeed, could not – be a number. Their basic strategy is to argue that the structure of their philosophical ontology is such that the problematic emperor is banned from the realm of numbers. In brief outline, the ontology is:

¹ In general, an abstraction principle is of the following form: $(\forall \alpha)(\forall \beta)(\Sigma(\alpha) = \Sigma(\beta) \leftrightarrow \alpha \approx \beta)$, where Σ is a term-forming operator taking expressions of the type of α and β as input, and \approx is an equivalence relation on entities denoted by expressions of that type.

² Championed by Bob Hale and Crispin Wright. Hale and Wright (2001a) is a collection of essays on the neo-Fregean programme.

³ Hale and Wright (2001b).

... a world in which all objects belong to one or another of a smallish range of very general categories, each of these subdividing into its own respective more or less general pure sorts; and in which all objects have an essential nature given by the most specific pure sort to which they belong. Within a category, all distinctions between objects are accountable by reference to the criterion of identity distinctive of it, while across categories, objects are distinguished by just that – the fact that they belong to different categories. It is surely because we already inchoately think in terms such as these that it strikes us as just obvious that Caesar is no number.⁴

In this paper I will do three things. First, I shall reconstruct Hale and Wright's solution to the Caesar Problem, which involves stating their characterization of the notion of a category. Second, I shall provide an alternative characterization of the notion of a category, which will make the structure of neo-Fregean ontology more clear – or at least this is the hope. Third, I will argue that the Caesar Problem can be solved in a framework more minimal than that of Hale and Wright, *viz.* one in which categories are dispensed with.

Before I pursue the tasks listed above, let me make explicit that the present paper will not be critical in the sense of attacking the good standing of the basic notions – *viz.* sortal concept and criterion of identity – underlying the Hale-Wright framework. Rather, as just stated, I will argue the point that the Caesar Problem can be solved in a framework which is slightly more minimal than that proposed by Hale and Wright, but nevertheless similar to it.

2. Hale and Wright's Solution to the Caesar Problem

In this section, a reconstruction of the Hale-Wright solution to the Caesar Problem will be given.⁵

Every object of the ontology falls under a sortal concept. The following principle is assumed to hold:

(P1) Every sortal concept has a unique criterion of identity.

The criterion of identity is provided by an equivalence relation that, for any two objects falling under the relevant sortal concept, settles the question whether they are identical or not. Thus, recalling Hume's Principle, for any two numbers of concepts, the relation of 1-1 correspondence serves as the criterion of identity.

When do sortals share their criterion of identity? Denote the relation of *sharing criterion of identity with* by " $S^=$." Hale and Wright adopt the following definition:

⁴ Hale and Wright (2001b), pp. 390-391.

⁵ Hale and Wright (2001b), Section 6. There is a bulk of literature on the notion of a sortal and the related notion of a criterion of identity. Certainly some authors differ greatly from Hale and Wright in their understanding of these notions. Here it is not our concern to discuss the merits of the various views on these matters. Throughout the paper we will assume a Hale-Wright view.

- (D1) For any two sortals X and Y , $S^{\bar{=}}(X, Y)$ if and only if it is conceptually necessary that any two objects are related by the equivalence relation giving the criterion of identity for X if and only if they related by the equivalence relation giving the criterion of identity for Y .

Note that $S^{\bar{=}}$ is an equivalence relation on sortals. It would not be, were P1 not adopted. Also, note that no special assumptions are made about conceptual necessity other than it should be such that $S^{\bar{=}}$ can be shown to be an equivalence relation.⁶

By *sortal inclusion* Hale and Wright mean the following:

- (D2) A sortal X is sortally included in a sortal Y if and only if, for every x , if x is X , then x is Y , and $S^{\bar{=}}(X, Y)$.

If a sortal X is sortally included in Y , we say that X is a *subsortal* of Y .

Among the sortals we find categories, which are maximally extensive sortals:

- (D3) A sortal X is a category if and only if (i) all sub-sortals of X share their criterion of identity, and (ii) for any object x , if it is not X , then any sortal Y which it falls under does not share its criterion of identity with X .⁷

The first condition is superfluous in light of D2, but is stated here to stay close to Hale and Wright's initial characterization. Furthermore, Hale and Wright need the following principles of existence:

- (P2) For any two sortals X and Y , if X and Y has one object x in common, there exists a sortal Z which is sortally included in X and sortally included in Y .
- (P3) Given sortals X_1, \dots, X_{κ} (κ a cardinal) that share their criterion of identity, there is a sortal co-extensional with their union.⁸

P3 is needed to ensure the existence of categories.

⁶ Thanks are due to Volker Halbach for making this latter point during the discussion at the GAP.5 Conference.

⁷ Dummett (1993), p. 76; Hale and Wright (2001b), p. 389. It should be recorded that the actual wording of condition (ii), as given by Hale and Wright, is ambiguous between an existential and a universal statement. According to the former statement, if an object does not fall under category X , then there is *some* sortal under which it falls which does not share its criterion of identity with X . The stronger statement – employed in D3 – is clearly the intended reading.

⁸ As stated, P3 is restricted to sortals that share criterion of identity. If the result of taking arbitrary unions of sortals were a sortal, it might be argued that the neo-Fregean would be committed to a single super-sortal – i.e. the union of all sortals in the ontology. However, it is unclear what the criterion of identity of such a super-sortal would be. Would it be a disjunctive one given by a disjunction involving all the equivalence relations giving the criteria of identity for each of the individual sortals? It is by no means obvious that such a disjunctive criterion of identity would satisfy D1.

Once this picture is in place, the following result can be established by straightforward applications of the definitions and principles:

(R1) Any two categories X and Y are either co-extensional or have no object in common.⁹

If, as is reasonable, it is supposed that two distinct categories are not co-extensional, we get:

(R2) If two sortals X and Y are included in distinct categories C and D, X and Y have no object in common.

The reasoning is simple. The sortal concepts of Number and Person have distinct criteria of identity.¹⁰ So, Number and Person cannot be sortally included in the same category, i.e. their categories are distinct. Hence, these categories are not co-extensional, and thus, by R1, they have no object in common. Therefore, Caesar cannot be a number. More generally, no person can be a number – and more generally still, no object falling under a given category can be identical to an object falling under a distinct category.

There might be an awkward wrinkle, though. Consider the following principle:

(U) No object can belong to more than one category.¹¹

An obvious thought is that this principle is redundant since, within the Hale-Wright framework, it follows from R2, and so, from the other principles and definitions. Let me try to give an informal explanation of why U follows in the present framework. According to Hale and Wright, sortals are kept in place in categories by their respective criteria of identity, and, in turn, objects are rooted in exactly one category through the criterion of identity of the sortals which they fall under. This explains why U holds from the Hale-Wright perspective.

However, a stubborn opponent might maintain that the *negation* of U can be combined with the Hale-Wright approach. Let me rehearse what Hale and Wright have to say on the matter.¹²

A feature of their approach is that there can be no recognition of cross-categorical identities, even if it is supposed that such identities might obtain. There are no controls on purported identifications of an object from one category with an object from a distinct category, since assessments of identity statements are always made by appeal to a

⁹ Indeed, Hale establishes an equivalent of the result in his brief note Hale (ms.) He shows that for any categories X and Y, if X and Y are not co-extensional, then there is no y which is both X and Y.

¹⁰ Hale and Wright take it for granted that the criteria of identity for Number and Person are distinct. See Hale and Wright (2001b), p. 389.

¹¹ Hale and Wright (2001b), p. 393.

¹² Hale and Wright (2001b), pp. 393-394.

criteria of identity, and these are internal to categories. Once it has been granted that two objects fall under two distinct categories, there can be no judgement or recognition of their identity.

To adopt the Hale-Wright framework and maintain the negation of U would thus force a commitment to the view that cross-categorical identities obtain in a manner unrecognizable, as a matter of principle, in that very framework. In other words, for an identity statement “ $a = b$ ” – where “ a ” and “ b ” are taken to denote objects in distinct categories – there can be no judgement, because there is no criterion of identity by appeal to which such a judgement could be made.

According to Hale and Wright, taking this line brings on a general indeterminacy concerning the reference of singular terms. The generality aspect of the indeterminacy is crucial. The Caesar Problem was raised as a worry concerning the ability of abstraction principles to adequately introduce sortal concepts. Hale and Wright granted that *on its own* an abstraction principle cannot adequately introduce a sortal concept as it gives no criterion of application. However, they developed an ontology based on principles concerning the notions – those of sortal concept and criterion of identity – taken to be central to abstraction principles from a neo-Fregean perspective and argued that, in this ontology, there could be no imperial inhabitants in the realm of numbers. The classifications and identifications assessable by the machinery provided by the neo-Fregean give determinacy in the sense reflected by R1, R2, and U. To insist on the negation of U is thus to insist on a kind of indeterminacy, *the source of which must be independent of the neo-Fregean machinery*. This resolves the Caesar Problem, conceived as a worry aimed specifically at the neo-Fregean position.

The situation invites a dilemma: the opponent must either grant the neo-Fregean that the Caesar Problem has been solved, or acknowledge that the indeterminacy involved is of a general kind, and thus not traceable to anything specific to the neo-Fregean framework. Either way, Hale and Wright maintain, the Caesar Problem has gone away.

3. An Alternative Characterization of the Notion of a Category

I shall now define the notion of a categorical sortal and state a result which shows it equivalent to the Hale-Wright notion of a category. My hope is that the alternative characterization will serve to make the structure of neo-Fregean ontology more transparent by describing it as constituted by equivalence classes of sortals under S^- .

Define the notions of an equivalence class under S^- and a categorical sortal as follows:

- (D3*a) The equivalence class of a sortal X under S^- is the class of sortals X is related to under the relation.
- (D3*b) A sortal X is a categorical sortal if and only if, for any x , x is X just in case x falls under a sortal in the equivalence class of X under S^- .

D3*a incorporates a fact stated above, namely that $S^=$ is an equivalence relation on sortals. D3*b basically says that X is a categorical sortal just in case it is co-extensional with its own equivalence class under $S^=$. P3 and D3*b ensure the existence of categorical sortals if they are fed equivalence classes under $S^=$.

The combination of D3*a and D3*b has the advantage of making it explicit how sortals bundle up in equivalence classes under $S^=$, and that categories – or categorical sortals, as it were – are intimately related to these equivalence classes.

Now, the following result can be established:

(R3) For any sortal X, X is a category if and only if X is a categorical sortal.¹³

An immediate corollary from R1 and R3:

(C1) Any two categorical sortals X and Y are either co-extensional or have no object in common.

and another one from R2 and R3:

(C2) If two sortals X and Y are included in distinct categorical sortals C and D, X and Y have no object in common.

4. Solving the Caesar Problem without Categories

I will now argue that the Caesar Problem can be solved without appeal to categories. So, suppose that we adopt D1, D2, D3*a, P1, and P2, but drop D3*b and P3. That is, we adopt a framework in which the notion of a categorical sortal – or category – is not defined, and correspondingly, the principle that ensures the existence of categorical sortal not assumed.

Why does this more minimal framework hold a solution to the Caesar Problem? The reason is this: maximal extensiveness is not essential to the proposed solution. What matters is that $S^=$ is an equivalence relation on sortals. Once this relation is in place, sortals bundle up in equivalence classes under the relation. Any two equivalence classes under $S^=$ will either be identical or disjoint, i.e. contain the exact same sortals or have no sortal in common. Since criteria of identity are assumed to be unique, no sortals with distinct criteria of identity can be in the same equivalence class under $S^=$.

Recall R2: if two sortals X and Y are included in distinct categories C and D, X and Y have no object in common. As said above, Hale and Wright take it for granted that the sortal concepts of Number and Person have distinct criteria of identity, and so, are sortally included in distinct categories. In our trimmed-down version of the framework,

¹³ I do not include a proof of this result. It can, however, be shown by straightforward applications of the definitions and principles.

we are not entitled to hold this, because we have left categories behind. We are, however, entitled to the following result:

(R2*) If two sortals X and Y are in distinct equivalence classes under S^- , X and Y have no object in common.

– which is the natural analogue of R2 in the more minimal setting.

To see why we are entitled to R2* reason as follows: suppose, with Hale and Wright, that the sortal concepts of Number and Person have distinct criteria of identity. By D3*a, Number and Person are in different equivalence classes under S^- . Now, recall from our discussion of U that, from the perspective of the neo-Fregean, candidates for identity qualify as such by there being a criterion of identity by appeal to which the relevant identity statement can be assessed. In cases of cross-categorical identities no such criterion is available, indeed could not be available.

Of course, strictly speaking, we cannot talk of cross-*categorical* identity statements in the economized framework. What we can do, however, is to preserve the basic thought that in order for objects to be candidates for identity there must be a criterion of identity in terms of which the relevant identity statement can be assessed. Because, once it has been granted, that Number and Person have distinct criteria of identity, by D3*a, these sortals are in different equivalence classes under S^- . This suffices to show – again, I stress, from a broadly neo-Fregean perspective – that there can be no criterion of identity, which can cater for an assessment of identity statements of the kind used to raise the Caesar Problem. That is to say, Caesar is not a candidate for being identical to any number. More generally, an object that falls under a sortal in one equivalence class under S^- is not a candidate for being identical to any object that falls under a sortal in a different equivalence class under S^- .

Consider the following principle:

(U*) No object can fall under sortals in two distinct equivalence classes under S^- .

– which is the counterpart of U in our trimmed down setting. As before, an opponent might insist that the economized framework presented above can be combined with the *negation* of U*, and so, that we have no solution to the Caesar Problem after all. However, the response will be similar to the one made in connection with the denial of U. The ensuing indeterminacy will be of a general kind and not to be blamed on anything specific to the economized neo-Fregean framework, and as such, the Caesar Problem can no longer be regarded as a worry only targeting the neo-Fregean position.

Again, as before, there will be a dilemma: the opponent must either grant that the Caesar Problem has been solved, or acknowledge that the indeterminacy involved is of a general kind. Either way, the Caesar Problem – conceived as specifically targeted at the adherent of the neo-Fregean framework – has gone away. Note, however, that this time no appeal to categories was made.

References

- Dummett, M. (1973): *Frege's Philosophy of Language*. London: Duckworth.
- (1993): *The Seas of Language*. Oxford: Oxford University Press.
- Hale, B. (ms.): Note on Caesar. Unpublished typescript.
- Hale, B. and C. Wright (2001a): *The Reason's Proper Study*. Oxford: Oxford University Press.
- Hale, B. and C. Wright (2001b): "To Bury Caesar...", in: Hale and Wright (2001b), pp. 335–396.