

Hume's Principle and entitlement:
On the epistemology of the neo-Fregean programme*

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Abstract

Hume's Principle (HP) is the key principle in the neo-Fregean account of arithmetical knowledge developed and defended by Bob Hale and Crispin Wright. In discussing the epistemology of the neo-Fregean programme, Wright has proposed that acceptance of HP is warranted non-evidentially, as a matter of entitlement. The aim of this paper is to discuss various issues that emerge in relation to the entitlement proposal. The issues divide into two groups. The first group of issues concerns entitlement in general and must be addressed by friends of entitlement regardless of context, while the second group of issues pertains specifically to the claim that HP is an entitlement.

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1 Introduction

An abstraction principle is a principle of the following form:

$$(\forall\alpha)(\forall\beta)(\Sigma(\alpha) = \Sigma(\beta) \leftrightarrow \alpha \sim \beta)$$

Here Σ is a term-forming operator that takes as inputs expressions of the type of α and β . \sim is an equivalence relation on entities denoted by expressions of this type.

According to the neo-Fregean, an epistemological foundation can be provided for classical mathematics by appeal to abstraction principles. This proposal has been developed most fully—and been most widely discussed—in the case of arithmetic, where the abstraction principle favoured by the neo-Fregean is what is known as *Hume’s Principle* (sometimes this will be referred to as ‘HP’ in the sequel):

$$(\forall X)(\forall Y)(\#X = \#Y \leftrightarrow X \approx_{1-1} Y)$$

That is, for any concepts X and Y , the number of X ’s is the same as the number of Y ’s if and only if there is a 1-1 correspondence between X and Y .¹

Hume’s Principle plays a crucial role in the proof of *Frege’s Theorem*, a result to the effect that Peano arithmetic is interpretable in the system obtained by adding the principle to pure second-order logic.² It is by appeal to this theorem that the neo-Fregean purports to be able to provide an epistemological foundation for arithmetic. In light of this, it is not surprising to find that much of the discussion of the prospects of a successful execution of the neo-Fregean programme has centered around the philosophical status of second-order logic and Hume’s Principle.³ This paper will follow the trend just noted and zoom in on Hume’s Principle, focusing on Wright’s proposal that acceptance of Hume’s Principle is warranted non-evidentially, as a matter of entitlement.

¹ $X \approx_{1-1} Y$ is definable in pure second-order logic: $X \approx_{1-1} Y =_{df}. (\exists R)(\forall x)(Xx \rightarrow (\exists y)(Yy \wedge (\forall z)(Rzx \rightarrow z = y))) \wedge (Yx \rightarrow (\exists y)(Xy \wedge (\forall z)(Rzx \rightarrow z = y)))$.

² See Wright [14] for details on the result and its philosophical significance. For further details cf. Hale and Wright [7], a collection of essays that develop the programme and respond to various criticisms launched against it.

³ Critical voices include Boolos [2] and [3], Dummett [4], Field [5], Fine [6], Heck [8], Rayo [9], Shapiro and Weir [11], and Weir [13].

2 Neo-Fregeanism and basic warrant

Before I move on to discuss entitlement, let me provide a bit of stage-setting to make clear why the good epistemic standing of Hume's Principle is of crucial importance to the neo-Fregean programme.

The neo-Fregean project is foundationalist in spirit. Arithmetical knowledge and warrant is to be accounted for by appeal to a set of basic principles, where by 'basic' is meant (at least) that the principles are non-inferentially warranted. For the neo-Fregean the set of basic principles—the foundation—is second-order logic supplemented by Hume's Principle. Part of the foundationalist package is a certain thesis about how warrant for belief in non-basic propositions is to be accounted for. Warrant to believe in non-basic propositions is acquired via inference from the basic principles, the idea being that acceptance of these principles enjoys a positive epistemic standing that gets transmitted to beliefs in non-basic propositions inferred from them.

Against the background of this picture the following question becomes very pressing:

(BASIC WARRANT)

How is acceptance of basic principles warranted?

This is a pressing question, because the warrant to accept the basic principles is what is supposed to account for the warrant to believe non-basic propositions. Absent a warrant to accept the basic principles, things will not get started on the transmission picture. Why? For the simple reason there will be nothing to transmit. Given the specific set of basic principles relied on by the neo-Fregean, (BASIC WARRANT) raises the following two questions:

1. How is acceptance of the principles of second-order logic warranted?
2. How is acceptance of Hume's Principle warranted?

Each question is important and interesting. However, as flagged above, the focus of this

paper will be on the second question.⁴ The neo-Fregean response to be explored below is that acceptance of Hume's Principle is warranted as a matter of entitlement. Wright's characterization of entitlement will be presented in Section 3. Some interpretative and clarificatory remarks will follow in Section 4.

It is worth emphasizing that there are certain things that should *not* be regarded as being part of the neo-Fregean project. In particular, the neo-Fregean attempt to provide an epistemological foundation for arithmetic brings on a commitment to neither the claim that Frege's Theorem is the way that people actually learn arithmetic nor to the claim that arithmetical reasoning is actually reasoning in pure second-order logic supplemented by Hume's Principle. Furthermore, the neo-Fregean is not out to vindicate some kind of 'justificatory imperialism' either. The neo-Fregean project is not meant to force the view that it is only via Frege's Theorem that one can acquire a warrant to accept the axioms of arithmetic. The thesis on offer is more modest: Frege's Theorem is *one* way in which a subject can come to learn the truth of the axioms of arithmetic. Accordingly, Wright characterizes the (epistemological part of the) neo-Fregean programme as an enterprise in 'reconstructive epistemology'.⁵

3 Entitlement of cognitive project

According to Wright, acceptance of Hume's Principle is warranted as a matter of entitlement. In his published work on neo-Fregeanism, Wright occasionally uses the term 'entitlement' explicitly to refer to a kind of warrant applicable to acceptance of Hume's Principle, but merely gives a gloss on the notion. The principle is supposed to be 'available without significant epistemological presupposition'⁶ or 'innocent until proven guilty'⁷. These glosses point to the key feature of entitlement, *viz.* that it is a default species of warrant. Hume's Principle is warranted unless it is shown to be otherwise.

⁴ The philosophical standing of second-order logic has received considerable attention, as a topic in the debate concerning the neo-Fregean programme as well as a topic in its own right. For a contribution to the former debate, see Shapiro and Weir [11].

⁵ Wright [20], Section I.

⁶ Wright [15], p. 279 and Wright [16], p. 321.

⁷ Wright [15], p. 282.

In more recent work on scepticism—and how best to respond to the challenge it poses—Wright characterizes entitlement in greater detail. The following characterization (or something like it—cf. fn. 8) is adopted:

Entitlement of cognitive project: Acceptance of a proposition P is an entitlement of a given cognitive project if

- (i) P is a *presupposition* of the project, i.e. if to doubt P in advance—or weaker: being open-minded about P —would rationally commit one to doubting (or being open-minded about) the significance or competence of the project;
- (ii) we have no sufficient reason to believe that P is untrue; and
- (iii) the attempt to justify P would involve further presuppositions in turn of no more secure a prior standing . . . and so on without limit; so that someone pursuing the relevant enquiry who accepted that there is nevertheless an onus to justify P would implicitly undertake a commitment to an infinite regress of justificatory projects, each concerned to vindicate the presuppositions of its predecessors.⁸

Let me say a little bit about how to understand the characterization.

We have cognitive projects pertaining to any subject-matter with respect to which we can undertake an investigation and *learn* things. That is, cognitive projects are projects whose successful execution amounts to a cognitive achievement. For instance, I

⁸ Wright [18], pp. 191–192. Note that the characterization presented here is not identical to the one presented by Wright—yet, it is a fair interpretation of what he has in mind. Let me highlight the differences between Wright’s own formulation and the one just given. First, Wright applies ‘entitlement’ to *propositions*. Entitlement is a species of warrant, and warrant is standardly conceived as something that applies to attitudes towards propositions rather than to propositions themselves. This is why the characterization given here is formulated in terms of acceptance. Several passages make it clear that Wright does see entitlement as something that applies to propositional attitudes rather than to propositions. Second, condition (i) differs from Wright’s own explicit characterization, which is phrased only in terms of doubt. Doubt is a stronger attitude than open-mindedness in the sense that doubt that P amounts to a positive attitude towards its negation, while open-mindedness involves a positive attitude towards neither P nor its negation. In various places, Wright implicitly relies on the modified characterization of a presupposition given here (e.g., Wright [18], p. 193).

might find myself wondering how big my suitcase is and learn that it is $19'' \times 14'' \times 8''$ when I measure it.

Let us turn to clause (i) and consider again the cognitive project of determining the dimensions of my suitcase. The proper functioning of my perceptual apparatus is a presupposition of this project. If I were to doubt (be open-minded about) about the proper functioning of my perceptual apparatus while undertaking the project, I would be rationally committed to likewise doubting (being open-minded about) the significance of the project itself. A presupposition of a cognitive project is thus an unavoidable commitment of that project, at least in the sense that doubting (being open-minded about) it rationally commits one to doubting (being open-minded about) its significance. Hence, the attitude held towards P must thus be one that excludes doubt (and open-mindedness).

Clause (ii) is a negative clause and captures the defining feature of entitlement—that it is a non-evidential species of warrant. An entitlement to accept P does not require the presence of positive evidence supporting P . Instead what is required is the absence of sufficient counter-evidence. We get a stark contrast when we compare my acceptance that my perceptual apparatus is functioning properly to my belief that my suitcase is $19'' \times 14'' \times 8''$. According to Wright, the former is warranted as a matter of entitlement. As just highlighted, this does not require any investigative work to be done. Entitlement—or non-evidential warrant—is warrant by default. On the other hand, if warranted, my belief that my suitcase is $19'' \times 14'' \times 8''$ is so in the sense of being justified. Justification, on Wright's view, is evidential warrant—and, so, the warrant for the belief in question does require investigative work to be done.

Clause (iii) is a regress clause. If it is supposed that acceptance of P is entitled, the clause tells us that attempts to acquire an evidential warrant, or a justification, to accept P must lead to an infinite regress. This regress will consist of justificatory projects that involve some presupposition of no more secure a prior standing than P itself. That is, a presupposition that is either of less or equally secure a prior standing. An example given

by Wright presents a regress that involves presuppositions of the same general kind.⁹ To illustrate return once more to the project of figuring out the dimensions of my suitcase and suppose that I am entitled to accept that my perceptual apparatus is functioning properly relative to this project. Now, suppose furthermore that I grant that there is an onus on me to provide a justification for this entitled acceptance—that is, to provide evidence in its favour. How might I proceed? Perhaps I will go to the doctor to have my hearing, eye sight, and so forth checked. The doctor’s findings might be communicated to me in a number of ways—verbally or in writing. The point now is that whichever way it is, acquiring the evidence that is supposed to deliver the justification draws in presuppositions of the very same kind as the one I set out to investigate. Whether I am told or read that my perceptual apparatus is functioning properly, I need to take it as a presupposition that my perceptual apparatus is functioning properly on that occasion. But that calls for a new investigation. And so on.

One more thing is worth noting about clause (iii). With a bit of unpacking, the clause can be taken to imply that no attempt to justify an entitlement P can improve its epistemic standing.¹⁰ It seems plausible to suppose that the following minimizing principle holds for justification (i.e. evidential warrant): a justification for a proposition cannot be any stronger than the weakest evidence supporting one of its presuppositions.¹¹ Metaphorically, if we think of the infinite regress as a ‘justificatory chain’, the minimizing principle says that justification for an entitlement P can be no stronger than the weakest link in its justificatory chain. Since any attempt to justify P will involve presuppositions of no more secure a prior standing, this means that attempts to justify P can bring no improvement in its epistemic standing. Here it will be useful to return to the example given above. As seen, the attempt to justify that my perceptual apparatus is functioning properly—a presupposition of one of my cognitive projects—involves presuppositions of

⁹ Wright [18], p. 189.

¹⁰ Here I have been helped by conversations with Carrie Jenkins.

¹¹ Wright [18], p. 191. Wright goes on to deny the principle in its full generality, the exceptions being presuppositions that are warranted non-evidentially. This is compatible with what has been said here since we are talking about justification, i.e. evidential warrant. Indeed, as far as I can tell Wright himself buys into the minimizing principle for justification.

the same general kind. These can be of no more secure prior standing than the initial presupposition. They are, after all, of the same general kind. Thus, if the attempt to justify P is supposed to improve on its epistemic standing, it must be concluded that it is bound to fail.

The above remarks should suffice to introduce entitlement of cognitive project. As already mentioned, Wright's proposal is that acceptance of Hume's Principle is warranted as a matter of entitlement. Certain issues are crucial to understanding this proposal fully. They are, nonetheless, issues that are left largely unaddressed by Wright. The objective of the remainder of the paper is to discuss some of these issues and further to raise a few critical points. As shall transpire, some of these issues or points pertain to entitlement in general, while others concern entitlement as applied specifically in the context of the neo-Fregean programme.

4 Hume's Principle and cognitive projects

Recall that acceptance of a proposition P qualifies as entitled relative to a given cognitive project. The first issue that I will turn to is what kind of cognitive project acceptance of Hume's Principle might qualify as an entitlement for. This is a basic matter that Wright does not say much about. However, the following passage does offer some pointers that put us in a position to table some qualified suggestions:

I argued that we have an entitlement of project to take that concepts are in good standing, absent specific reason to think the contrary. But our concepts in general are in good standing only if standard means for explaining them are effective—establish them as such. That standard basic means, whether it proceeds by explicit stipulation of ground for and consequences of judgements which configure a given concept, or whether it proceeds by immersive explanation, as it were, of practices in which those grounds and consequences are acknowledged, is implicit definition. So we are entitled to take it that implicit definition is a generally effective means of conceptual innovation and hence that, in the absence of specific reasons for misgivings, it is productive of concepts in good standing in particular cases. The success of any particular—statement or process of—implicit definition in con-

veying a concept in good standing should be the default assumption.¹²

Applying the above line of thought to Hume's Principle—the neo-Fregean's favoured principle for implicit definition in the case of arithmetic—one kind of cognitive project would appear to be the following:

Cognitive project 1: Hume's Principle is a presupposition of the project of introducing a concept of number in good standing. It is so because doubt or open-mindedness about Hume's Principle rationally commits one to doubt or be open-minded about whether the principle has any counterexamples—that is, whether there are cases in which two concepts stand in a 1-1 correspondence and yet their numbers are different, or cases in which there are numbers of concepts that are the same although there is no 1-1 correspondence between their concepts. Since Hume's Principle gives the criterion of identity for numbers, it pertains to something integral to the concept of number. Hence, a rational commitment to doubt or open-mindedness about the principle in effect amounts to a rational commitment to doubt or open-mindedness about the very prospects of introducing a concept of number in good standing.¹³

It seems difficult to deny that Hume's Principle is significant in the way just mentioned. So, the neo-Fregean should grant that acceptance of Hume's Principle is a presupposition of the project of introducing the concept of number. Are there other kinds of cognitive project for which acceptance of Hume's Principle qualifies as a presupposition? It would seem so:

Cognitive project 2: Acceptance of Hume's Principle is a presupposition of arithmetical projects. If one were to doubt or be open-minded about Hume's Prin-

¹² Wright [20], Section IV.

¹³ Things would go terribly wrong if the principle were to have counterexamples. A model in which Hume's Principle fails can be thought of as the analogue of a non-cooperative scenario in the case of scepticism about the empirical world, i.e. a demon/vat world.

ciple, one would likewise be rationally committed to doubt or being open-minded about the significance of any arithmetical project.

Cognitive projects 1 and 2 are related. The reason why acceptance of Hume's Principle is a presupposition of any arithmetical project whatsoever is traceable to its being a presupposition of attempts to introduce the concept of number. In order to introduce a concept of number in good standing one cannot doubt or be open-minded about Hume's Principle, for the reasons given above. But the significance of arithmetical projects is dependent on the availability of a concept of number in good standing. After all, arithmetic is the theory of (natural) numbers—and that theory will be no good unless its core concept is in good standing.

In sum, acceptance of Hume's Principle is a presupposition of a wide class of projects, including not just attempts to introduce a concept of number in good standing, but also any arithmetical project whatsoever. This is a desirable feature in that it makes entitled acceptance of Hume's Principle line up rather nicely with entitlement as it is applied in the discussion of scepticism about the empirical world. In the empirical world case the sceptical attack is of a high degree of generality. The propositions attacked by the sceptic—e.g. that I am not a brain in a vat or that I am not being deceived by an omnipotent, evil demon—are presuppositions of more or less all of our cognitive projects concerning the empirical world. Doubting or being open-minded about whether I am a brain in a vat would certainly seem rationally to commit me to doubting or being open-minded about any project I may have pertaining to the empirical world—determining the dimensions of my suitcase, say. Since acceptance of Hume's Principle is a presupposition of arithmetical projects in general, entitlement in the neo-Fregean setting will likewise be of wide epistemic significance relative to the domain under discussion.

5 Reasons

The previous section concerned the issue how to understand clause (i) on the specific assumption that acceptance of Hume's Principle is entitled. This section concerns the question what it takes for something to be a reason. This is relevant to understanding clause (ii) ('we have no sufficient reason to believe P untrue'). This issue does not just arise specifically in the context of the neo-Fregean programme, but pertains to entitlement in general. Below I will offer a common-place distinction between two conceptions of reasons—one metaphysical, the other epistemic—that leads to substantially different readings of clause (ii).

Metaphysical reasons.

On one conception of reasons—the metaphysical conception—what reasons a subject S has is determined by the world. The world determines the truth-value of propositions, and the set of propositions—with their world-determined truth-values—constitutes the domain of reasons relevant to clause (ii). This conception of reasons may strike some as slightly odd. However, it does capture a good notion of a reason, at least in the sense that it lines up with certain ways of thinking or talking about reasons. Let me illustrate by way of example. Suppose that Susan is out for a drive in her car and that she has just started making a turn. Furthermore, suppose that unbeknownst to Susan a kid—who is completely oblivious to her surroundings—is playing in the street just around the corner. In this case it seems to make perfect sense to say that Susan has a reason to hit the brakes. What gives Susan a reason to hit the brakes here is the fact that there is a kid in the street, i.e. the world's being in a certain way.

The following feature of the metaphysical conception is worth highlighting: if it is assumed that every proposition P is either determinately true or determinately false, no false proposition Q can qualify as an entitlement. For if Q is false, it means that $\neg Q$ is true (assuming classical negation)—and the truth of $\neg Q$ is a sufficient reason to believe Q false.

Epistemic reasons.

According to the epistemic conception of reasons, what reasons a subject has for adopting a certain doxastic attitude towards a proposition is given by the set of propositions that are within her epistemic, or cognitive, reach. The propositions that are epistemically reachable by *S* are those propositions that *S* would hold some doxastic attitude towards were she to exercise her cognitive resources fully (where by ‘cognitive resources’ is meant *S*’s current capacities, abilities, skills, methods, and procedures).

Note that, running along these lines, the domain of reasons that is relevant to clause (ii) will exclude propositions that *S* would never consider—e.g. those propositions she does not possess the conceptual resources to consider. Also, whether clause (ii) is satisfied is independent of *S*’s *current* attitudes to the cognitively reachable propositions. What is relevant is what attitudes *S* would hold if she were to exercise her cognitive resources fully. Furthermore, it should be observed that a candidate entitled acceptance of *P* might be true, and yet be undermined by a false proposition (i) which would be believed by *S* after fully exercising her cognitive resources and (ii) which suffices for thinking that *P* is untrue. On the other hand, it might be that a candidate entitled acceptance of *Q* is false, but satisfies clause (ii), because the domain of reasons contains no proposition whose truth suffices for believing *Q* false. This is a respect in which the epistemic conception of reasons differs from the metaphysical conception.

The metaphysical and epistemic conceptions of reasons make different domains of reasons relevant to clause (ii). These domains are strikingly different. On the metaphysical conception, the domain of reasons is determined by what *is* fact, while, on the epistemic conception, the domain of reasons is determined by what is taken to be fact.

6 Defeasibility

Wright intends entitled acceptances to be defeasible. They can be possessed at a given time, but lost later. This is due to clause (ii). It is meant to be understood in such a way that there might initially be no sufficient reason to think some proposition P untrue, and yet later such a reason might be present—in which case the satisfaction of clause (ii) is undermined. Unfortunately, neither of the two conceptions of reasons from the previous section supports a reading of clause (ii) that makes entitled acceptances defeasible. I will first show why and then proceed to discuss if there is any way to accommodate defeasibility.

Let us turn first to the metaphysical conception of reasons. This conception commits one to the non-defeasibility of entitled acceptances, at least given two assumptions. Supposing that R is the region of thought that we are concerned with, the first assumption—bivalence—is this: every R -proposition is either determinately true or determinately false. The second assumption is that the truth-value of any R -proposition is stable, i.e. that it does not change over time.¹⁴

Consider a candidate entitled acceptance of P against the background of the metaphysical conception and these two assumptions. Recall that we get defeasibility if acceptance of P can be entitled at one point, but fail to be so at some later point. As highlighted earlier, only acceptance of true propositions can qualify as entitled on the metaphysical conception of reasons. So, suppose that P is true and its acceptance entitled—meaning, in particular, that clause (ii) is satisfied. By the stability assumption P will remain true. This means that it will remain the case that the world is such that nothing is sufficient to making P false. Since reasons for belief on the metaphysical conception are simply given by the facts—or the state of the world—this means that there is no sufficient reason to believe P untrue. That is to say, there is nothing to undermine the satisfaction of clause (ii) and render entitled acceptances defeasible.¹⁵

¹⁴ Although the remarks to be offered on defeasibility are general in nature, it is worth noting specifically that the two assumptions at hand are ones that should be granted in the context of a discussion of the neo-Fregean programme.

¹⁵ Let me say a little bit about what work the bivalence and stability assumptions are doing here. I will

Let us now consider the epistemic conception of reasons and see if this conception can support defeasibility. Suppose that S is entitled to accept P relative to some domain of reasons D_R and a fixed class of methods and procedures CM . Recall that D_R includes every proposition Q which S would believe if she were to investigate the question whether Q by fully exercising her capacities together with the methods or procedures of CM . Thus, the status of acceptance of P as an entitlement is determined by reference to the best qualified beliefs S can (counterfactually) come to hold given her current cognitive repertoire. In this sense there is no room for improvement or fluctuation with respect to what propositions are in D_R . For this reason entitled acceptance will be non-defeasible. Whether, in the epistemic sense, there is sufficient reason to believe a candidate entitlement P untrue is determined by D_R , and since D_R is stably fixed, so is the question whether or not clause (ii) is met.

In sum, we have that neither the metaphysical conception nor the epistemic conception of reasons supports a reading of clause (ii) that renders entitled acceptance defeasible. Or at least neither conception does so if we understand the conceptions in the way that we have thus far. I will now turn to the question whether there is any way to modify either conception so as to yield a reading of clause (ii) that supports the intended defeasibility of entitled acceptances. It turns out that there is. Below I present

do so by briefly discussing the domain a law, a domain where these two assumptions seem implausible.

Most systems of laws are incomplete. They typically fail to determine completely what is and what is not lawful. Not every legal proposition is determinately true or determinately false. Without determinacy it can be indeterminate whether or not there is any sufficient reason—understood in the metaphysical sense—to believe a given proposition P untrue. It might be that there are lots of propositions whose truth would imply the falsity of P and, so, would be sufficient reasons to believe P untrue. However, it may be that it is indeterminate whether these propositions are true. In that case the metaphysical reading of clause (ii) supports neither defeasibility nor non-defeasibility.

Let me now turn to the stability assumption. The law is what determines the truth-value of legal propositions. However, it does not do so stably. For the law is mind-dependent and is subject to change. Specific laws can be amended or otherwise modified, or dropped. New laws can be added to the existing law. Suppose that current law says that some legal proposition P is true. Well, the law-makers can change the law so P is not true. Today's crime might not continue to be a violation of the law, or the other way around. Without stability the metaphysical reading of clause (ii) could render entitled acceptances defeasible. Consider an entitled acceptance of P . Since acceptance of P is entitled, there is no sufficient reason to believe a proposition P untrue—meaning, on the metaphysical reading of clause (ii), that the world is in such a way that there is nothing sufficient to make P untrue. Yet, at a later time it might be that some proposition Q is made true by the law due to some change or addition, and that the truth of Q suffices for the falsity of P . In that case the satisfaction of clause (ii) would be undermined, and the entitled acceptance of P defeated.

two ways in which this can be accomplished for the epistemic reading of the clause.

Assume that the epistemic conception of reasons is taken as background for understanding clause (ii). Then either of the following two modifications will support defeasibility:

- (1) Allow clause (ii) to be satisfied relative to what propositions the subject would hold some attitude towards if she were to exercise her cognitive resources to a certain degree, but undermined if the subject were to exercise those very resources more carefully or to a greater degree. (This degree, although greater, might fall short of fully exercising the resources in question.)
- (2) Allow clause (ii) to be satisfied relative to what propositions the subject would hold some attitude towards if she were to exercise a fixed set of cognitive resources to a certain degree (full or not), but undermined if the subject were to exercise an extended, improved set of resources to the same degree.

Let me offer a few examples to shed light on (1) and (2).

I turn to (1) first. Consider Frege's *Grundgesetze*. It seems that Frege was entitled to Basic Law V in doing arithmetic. He was so, because (i) Basic Law V was a presupposition of arithmetical projects within Frege's system¹⁶, (ii) after carefully exercising his cognitive resources Frege did not hold any doxastic attitudes that presented a sufficient reason to believe Basic Law V untrue, and (iii) granting an onus to justify—i.e. acquire an evidential warrant for—Basic Law V would commit one to undertaking an infinite regress of justificatory projects involving presuppositions of no more secure a prior standing than Basic Law V. As is well-known, Frege received a letter from Russell that led him to realize—and believe—that Basic Law V generates an inconsistency in the system of *Grundgesetze*. Although in the actual course of events it was Russell's letter that prompted Frege to arrive at this belief, Frege's cognitive resources were still such

¹⁶ In the framework of Frege's *Grundgesetze*, Basic Law V stands much to arithmetic as HP stands to arithmetic in the neo-Fregean programme. In other words, Basic Law V is crucial to the good standing of the concept of number.

that he might have come to hold the belief if he had exercised his cognitive resources more carefully on his own, even if less than fully. In such cases clause (ii) would go from being satisfied to being undermined, meaning that the modified conception of epistemic reasons can accommodate the defeasibility of entitled acceptance.

As for (2), suppose that acceptance of P qualifies as entitled relative to a set of propositions which subject S would hold a doxastic attitude towards if she were to exercise her *current* cognitive resources fully. It might be that there are certain propositions that S has never considered due to limitations in her current cognitive resources, but which she would hold some attitudes towards if her cognitive resources were to be improved in relevant respects by, say, the addition of new methods or procedures. It might also be that there are certain propositions that S holds some specific attitude towards, but which she would hold a different attitude towards provided that her cognitive resources were improved in relevant respects. In both of the cases just envisioned it might be that the domain of reasons determined by S 's improved cognitive resources contains a sufficient reason for believing P untrue, and in this sense the entitlement to accept P can be said to have been defeated.

Again, let me supplement the point with an example. The example is derived from the Basic Law V scenario by making some small changes. Let Frege* be someone very much like Frege in that, just like Frege, Frege* is working in a system—let it be *Grundgesetze**—that he takes to provide a basis for arithmetic, and which features Basic Law V* among its central principles. Let us also suppose that Frege* has an entitlement to accept Basic Law V*, and that Frege* would not believe anything sufficient to believe Basic Law V* untrue if he were to exercise his current cognitive capacities fully. However, consider now another subject, Russell*, who is able to derive a contradiction from Basic Law V* in the context of the system of *Grundgesetze**. He does so by using a new method he has developed—one that is not in Frege*'s cognitive repertoire. In this case there is an improved set of resources—one that includes Russell*'s new method—that would lead Frege* to believe that Basic Law V* generates an inconsistency in his system. This belief would defeat Frege*'s entitlement to accept Basic Law V*. Hence, (2)—just

like (1)—offers a way in which the epistemic conception can be modified so as to render entitled acceptance defeasible.

7 Sufficiency

In the preceding sections we have discussed two issues pertaining to reasons—firstly, what conceptions of reasons are available when it comes to understanding clause (ii), and secondly—and relatedly—whether any of them can support the intended defeasibility of entitled acceptance. In this section, I turn to yet another issue regarding reasons. The issue is this: what exactly does it mean for a reason to be *sufficient* for believing a proposition P untrue? This is a fair question to ask whether the metaphysical or some epistemic conception of reasons is adopted. For on any of these conceptions addressing the issue will clearly be crucial to understanding clause (ii).¹⁷

Whether the metaphysical or some epistemic conception is adopted, there are clear cases in which there is a sufficient reason to believe something untrue. On the metaphysical conception, the fact that Basic Law V is inconsistent is a sufficient reason to believe it untrue, and so, a reason why acceptance of Basic Law V cannot be entitled when clause (ii) is given a metaphysical reading. On an epistemic reading of clause (ii)—with the epistemic conception modified along the lines of (1) of the previous section—Basic Law V would not satisfy clause (ii) either. For if Frege had exercised his cognitive resources more carefully, he would have believed that Basic Law V is inconsistent.

Are there cases where the verdict is less clear? Yes, there are. I will present such a case now. In doing so I will take the epistemic conception of reasons as background and, accordingly, will presuppose an epistemic reading of clause (ii).

Consider the following scenario: Anti-Hero is wondering whether some abstraction principle AP is inconsistent in the context of standard second-order logic. If Anti-Hero were to think more carefully about the matter, he would be in a position where he would have a proof sketch of a theorem to the effect that AP is indeed inconsistent. Let us

¹⁷ I am indebted to Patrick Greenough and Graham Priest for raising this point in conversation on separate occasions.

suppose that the sketch would have four lemmas, and that the sketch would be pretty detailed for three of the lemmas, but give no details are added for the fourth. In this case it would seem unclear whether Anti-Hero has a sufficient reason to believe AP untrue.¹⁸

Someone might object that, as described, the scenario is not very specific and insist that this makes it difficult to say whether or there is sufficient reason to believe AP untrue. That might be so. Still, it seems plausible to me that there are cases like it with the relevant details added and in which it is unclear whether there is a sufficient reason to believe some proposition P untrue. For such cases the unclarity as to whether there is sufficient reason to believe P untrue would not be due to a lack of detail—all the details would be there—but rather due to the lack of a criterion of sufficiency for reasons.

It should be clear that the issue just raised pertains to entitlement in general. However, it should likewise be clear that a neo-Fregean like Wright ought to address it in so far as he wants to rely on entitlement in telling a story about how acceptance of Hume's Principle might be warranted.

8 The Generosity Problem

In this section I will discuss another issue pertaining to entitlement in general. We can get at the issue by looking at the following passage from Wright:

This is a good result [that acceptance of anti-sceptical propositions is entitled], it goes without saying, only if it is selective—only if the entitlements generated turn out to be cornerstones of our actual ways of thinking about and investigating the world and do not extend to all manner of irrational and bizarre prejudices.¹⁹

The issue tabled here is that entitlement might be too generous a notion. Entitlements might not, as Wright worries, be sufficiently selective but 'extend to all manner of ir-

¹⁸ The point just made rests on the assumption that the epistemic conception of reasons is understood along the lines of (1) of the previous section. However, it would take only little modification to adjust the case to make it work for the version of the epistemic conception that is paired with the subject's fully exercising her cognitive resources.

¹⁹ Wright [18], p. 195.

rational and bizarre prejudices’ rather than apply exclusively to ‘cornerstones of our actual ways of thinking about and investigating the world’. Wright holds that the usual anti-sceptical propositions—that I’m not a brain in a vat, among others—fall in the latter category. However, what falls into the first category, the category of irrational and bizarre prejudices? Wright invites us to consider the following case:

Suppose I postulate a tract of reality—it might be the realm of non-actual possible worlds as conceived by Lewis—which is spatio-temporally insulated from the domain of our usual empirical knowledge, and a special faculty—as it may be, our non-inferential ‘modal intuition’—whose operation is supposed to allow us to gather knowledge about it.²⁰

Is acceptance of the reliability of modal intuition entitled relative to the region of ‘modalizing’? (Or, as Wright says, ‘our primitive, non-inferential impressions of modal validity and invalidity.’) According to Wright it might very well be:

After all, I have—in the nature of the case, since I cannot compare its deliverances with the facts, independently ascertained—no reason to believe that it is unreliable (so long as its promptings are consistent); and any attempt to check on its functioning will presumably perforce to involve further modal intuition, ‘of no more secure a prior standing.’²¹

Let me make an attempt to add a bit of detail. Accepting that modal intuition is reliable is a presupposition of modalizing, because modal intuition is supposed to be the only means by which the realm of possible worlds can be investigated. Accordingly, doubt about the reliability of modal intuition would rationally commit one to doubting the significance of whatever modal project one might engage in. So, clause (i) is satisfied.

Also, it would seem that there is no sufficient reason to think that modal intuition is unreliable—where this is understood in an epistemic sense. Considering *individual* pieces of modal information is not something that could lead us to believe high enough a

²⁰ Wright [18], p. 196.

²¹Wright [18], p. 196.

proportion of them false. This is because, by assumption, we have no independent grip on the realm of possible worlds which can serve as an external standard against which we can measure the correctness of individual deliverances of modal intuition. Matters change somewhat if we consider sets of pieces of modal information. For in that case, as Wright indicates, one way that there could turn out to be sufficient reason to think modal intuition unreliable would be if we were to believe a high percentage of pieces of modal information to be inconsistent with each other, upon carefully or fully exercising modal intuition and other relevant capacities (such as a priori reflection). However, the point is that this does not seem likely. Clause (ii) thus also appears to be satisfied.

As for clause (iii), attempts to vindicate the reliability of modal intuition must give rise to an infinite regress of justificatory projects that involves some presupposition of no more secure a priori standing than the reliability of modal intuition itself. Now, to vindicate the reliability of modal intuition one would have to consider its deliverances and justify the claim that high enough a proportion of them are true. Yet, as emphasized earlier, modal intuition is the only means by which we can access the relevant realm. Hence, testing the reliability of modal intuition would inevitably bring on a need to presuppose the reliability of modal intuition—that is, a need to presuppose exactly what was meant to be vindicated in the first place.

Clauses (i)-(iii) are all satisfied. As a result, accepting that modal intuition is reliable should qualify as being entitled. Indeed, generalizing on the case at hand, it would appear that we have an instance of entitlement whenever we postulate (i) a tract of reality R which does not interact in any way with domains accessible through our usual cognitive powers or methods, and (ii) a corresponding special faculty or method—the R -faculty or R -method—by means of which we can access the tract of reality in question. It is a presupposition of R -projects, as doubt about its reliability will rationally commit one to doubting the significance of the R -project. Likewise there is no sufficient (epistemic) reason to think it untrue that the R -faculty is reliable, and attempts to vindicate its reliability will lead to an infinite regress of justificatory projects involving some presupposition of no more secure a priori standing than the reliability of the R -

faculty itself. Hence, acceptance of the reliability of the special *R*-faculty seems to be entitled for cognitive projects pertaining to the postulated tract of reality. The reasoning from the modal case can be modified to support this more general conclusion.

Now, let us return to the modalizing case for a moment. Does the entitlement to accept the reliability of modal intuition fall under the heading of ‘irrational and bizarre prejudices’ rather than being a cornerstone of our actual ways of thinking about the world? Arguably not. Modal reasoning is part of our actual thinking about the world. However, one *can* generate the kind of unwanted entitlements that Wright worries about by using the template extracted from the modal case.

Suppose that Mr. X believes in (i) a tract of reality, Pixie World, inhabited by pixies and completely isolated from domains which we can gain access to by our ordinary faculties or methods, and (ii) a special faculty, the Pixie Eye, detecting the whereabouts and doings of the pixies. Suppose, furthermore, that Mr. X’s overall conception of the world—including (i) and (ii)—is consistent. When investigating Pixie World, is Mr. X entitled in accepting that the Pixie Eye is a reliable faculty? The case fits the generalized template presented above. This suggests that Mr. X’s acceptance is indeed entitled—and, surely, this entitlement is not part of our actual ways of thinking about the world. Rather, it must be deemed crazy or bizarre from our perspective.

In light of the above, entitlement seems to apply quite generously—indeed, to any acceptance that complies with (i) and (ii) of the generalized template. I am inclined to regard this generosity not as a problem for entitlement, but rather as a feature. However, Wright has to reject this idea to stay true to the thought that entitlement should be sufficiently selective. This leaves him with the task of spelling out why the would-be troublemakers—the ‘irrational or bizarre prejudices’—might have the appearance of being entitlements, but fail to really be so in the end.

9 Hume's Principle and relative consistency

I have discussed various issues that concern entitlement in general. I will now turn to a worry targeted specifically at the idea that acceptance of Hume's Principle is entitled. To be in a position to spell out the worry we need to return to clause (iii) and also have a look at the following two passages from Hale and Wright:

... as far as the crucial question of consistency is concerned, we now have as much assurance as it seems reasonable to demand. (Hale and Wright [7], p. 5)

... A little more precisely, what Boolos shows is not just that we can give models for Hume's Principle ..., but that the informal model-theoretic proofs can be replicated not only in standard set theory, but in the weaker theory known as 'second-order arithmetic' or 'analysis', the consistency of which seems well beyond serious question. So while these consistency proofs for FA [i.e. Frege Arithmetic, which amounts to Hume's Principle plus second-order logic] are, strictly speaking, merely relative consistency proofs, they nevertheless constitute a very strong result. (Hale and Wright [7], p. 6)

The worry I want to raise is whether, in light of the above passages, acceptance of Hume's Principle can be really be said to satisfy clause (iii). In order for this worry to get a grip the passages just cited need to be given an epistemological reading. That is, the relative consistency proof of which Hale and Wright speak should be regarded as having epistemological punch. One might think that it does, because the consistency of FA—including Hume's Principle—is well beyond serious question, while the consistency of analysis is. The strong standing of analysis is thus meant to boost the epistemic standing of acceptance of Hume's Principle.

Let us now turn to clause (iii), which—when applied to Hume's Principle—requires that attempts to justify acceptance of the principle give rise to an infinite regress involving some presupposition of no more secure a priori standing than the principle itself. As seen in Section 3, when unpacked properly clause (iii) carries with it a commitment to the thought that entitled acceptance of P is a standing that cannot be improved. Just to

repeat, recall that in order for clause (iii) to be satisfied attempts to justify acceptance of P must lead to an infinite regress of justificatory projects involving presuppositions of no more secure a priori standing than P itself. That is, there will be at least one proposition in the ‘justificatory chain’ of a lesser epistemic standing than P , or one that is equal to it. Paired with the minimization principle for justification this yields the result that P ’s epistemic standing can never be improved through execution of the justificatory projects in the regress.²² In particular, if acceptance of Hume’s Principle is entitled, it should not be possible to improve its epistemic standing. But this sounds slightly puzzling in light of the epistemic reading of the quoted passages, according to which the proof of FA’s consistency relative to analysis is meant to do exactly that, i.e. to boost the epistemic standing of FA, including Hume’s Principle.

Bracketing for a moment the proposed epistemic reading, there is one sense in which the talk of relative consistency proofs makes for a happy fit with clause (iii). I have in mind the following: in showing FA consistent relative to analysis the consistency of FA is held hostage to the consistency of analysis. Whatever its merits, the relative consistency proof does nothing to establish *that*. Due to Gödel’s second incompleteness theorem, the best one can do with respect to the consistency of analysis—indeed, any theory strong enough to express elementary arithmetic—is to establish it relative to some other theory T of consistency strength greater than that of analysis itself. Thus, the consistency of analysis will be held hostage to that of T . The pattern repeats itself, and a regress of relative consistency proofs involving stronger and stronger theories results. This seems to fit well with clause (iii), at least if consistency strength is a measure of epistemic standing, and consistency strength and degree of epistemic security are inversely proportional. For in that case the regress that emerges will involve presuppositions that are of no more secure a priori standing than Hume’s Principle itself—*viz.* the stronger theories.

However, satisfaction of clause (iii) along the lines just sketched clashes with the epistemic reading of the passages highlighted. For, if consistency strength and degree of

²² The minimization principle was that a justification for a proposition cannot be any stronger than the weakest evidence supporting one of its presuppositions. Intuitively, the minimizing principle says that justification for an entitlement P can be no stronger than the weakest link in its justificatory chain.

epistemic security are inversely proportional—and so, higher consistency strength means a lower degree of epistemic security—there would be something odd about claiming that showing FA consistent relative to analysis can boost the epistemic standing of acceptance of FA, and Hume’s Principle in particular. By the inverse proportionality of consistency strength and degree of epistemic security, analysis should enjoy less secure a standing than FA. How then should the relative consistency proof in question improve the epistemic standing of acceptance of Hume’s Principle? It should not.

It has been argued that the epistemic reading of the passages cited from Hale and Wright’s clashes with the assumption that clause (iii) is satisfied. It does so firstly if we simply recall that clause (iii) brings on a commitment to the idea that entitled acceptance is a standing that cannot be improved upon. Secondly, the epistemic reading also clashes with clause (iii) if the clause is taken to involve an infinite regress of relative consistency proofs with theories of increasing strength *and* consistency strength and degree of epistemic security are taken to be inversely proportional. In light of this, the natural suggestion regarding the passages in question is that the neo-Fregean cannot adopt the epistemic reading. This leaves the neo-Fregean with the task of showing what, given the unavailability of the epistemic reading, should be made of the comments on relative consistency.

10 Conclusion

Hume’s Principle plays a crucial role in the neo-Fregean programme. This paper has been concerned with the epistemology of Hume’s Principle, especially Wright’s proposal that acceptance of the principle is warranted non-evidentially, or as a matter of entitlement. My hope is to have accomplished two things in this paper. First, I hope to have managed to shed some light on the entitlement proposal by presenting and discussing several fundamental issues pertaining to the proposal. Some of these were general, others specific to the neo-Fregean setting. Second, I hope that my discussion of these various issues have made it clear that there are still some very substantial questions and po-

tential worries that friends of entitlement need to attend to, generally as well as in the specific context of the neo-Fregean programme.

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